Cylindrical and spherical ion-acoustic envelope solitons in multicomponent plasmas with positrons

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The nonlinear wave modulation of planar and nonplanar (cylindrical and spherical) ion-acoustic envelope solitons in a collisionless unmagnetized electron-positron-ion plasma with two-electron temperature distributions has been studied. The reductive perturbative technique is used to obtain a modified nonlinear Schrödinger equation, which includes a damping term that accounts for the geometrical effect. The critical wave number threshold K_c, which indicates where the modulational instability sets in, has been determined for various regimes. It is found that an increase in the positron concentration (α) leads to a decrease in the critical wave number (K_c) until α approaches certain value α_c (critical positron concentration), then further increase in α beyond α_c increases the value of K_c. Also, it is found that there is a modulation instability period for the cylindrical and spherical wave modulation, which does not exist in the one-dimensional case.

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I. INTRODUCTION

An electron-positron plasma, a fully ionized gas composed of electrons and positrons having equal masses and charges with opposite polarity, is considered not only as a building block of our early universe [1], but also as an omnipresent ingredient of a number of astrophysical objects, such as active galactic nuclei [2], pulsar magnetospheres [3], solar flares [4], fireballs producing γ-ray bursts [5], etc. Electron-positron plasmas are also observed in laboratory experiments in which the positrons can be used to probe the particle transport in tokamak plasmas [6–8]. Processes of electron-positron pair production can occur during intense short laser pulse propagation in plasmas [9]. However, because of the rather long lifetime of positrons, most of the astrophysical [1,4,5] and laboratory [6–8] plasmas become an admixture of electrons, positrons, and ions. Surko and Murphy [10] have reported that over a wide range of parameters, annihilation of electrons and positrons, which is the analog of recombination in plasma composed of ions and electrons, is relatively unimportant. They have also reported that even at an electron density of 1×10^{12} cm^{-3} and a temperature as low as 1 eV, the positron annihilation time is greater than 1 s. Therefore, the study of electron-positron-ion (EPI) plasmas is important to understand the behavior of both astrophysical and laboratory plasmas.

Recently, the wave propagation in such a three-component EPI plasma has attracted much interest; see for example Refs. [9–18]. Rizzato [11] considered weakly nonlinear circularly polarized electromagnetic waves in a cold EPI plasma with stationary ions. Berezhiani *et al.* [9,12] investigated the nonlinear propagation of intense electromagnetic radiation in a magnetized EPI plasma. Rizzato [11] and Berezhiani *et al.* [9,12] found that such a three-component plasma supports radiation driven humped electrostatic potentials, which can be used to accelerate charged particles. Ion-acoustic solitary waves have been studied in an unmagnetized three-component EPI plasma [13]. The nonlinear investigation showed that the amplitude of the electron density hump reduces due to the presence of positrons in the electron-ion plasma. Yu *et al.* [14] studied inertial Alfvén solitary vortices in a strongly magnetized pair plasma. Double layers associated with the kinetic Alfvén waves in a magnetized EPI plasma have been studied by Kakati and Goswami [15]. The nonlinear propagation of ion-acoustic waves in EPI with trapped electrons has been studied by Alinejad *et al.* [16]. Tiwari *et al.* [17] studied the effects of positron density and temperature on ion-acoustic dressed solitons in EPI plasma. Sabry *et al.* [18] investigated for the nonlinear structures (explosive, solitons, and shock) in quantum EPI magnetoplasmas.

Two-electron temperature distributions are very common in the laboratory [19,20], as well as in space plasmas [21]. Shatashvili *et al.* [22] have reported that out flows of the electron-positron plasma from pulsars entering an interstellar cold, low-density electron-ion plasma form two-temperature EPI plasma. Mishra *et al.* [23] investigated ion-acoustic

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double layers in EPI plasma with two-electron temperature distributions. They found that there exist two critical concentrations of positrons, which decide the existence and nature of the ion-acoustic double layers.

Recently, Salahuddin *et al.* [24] investigated ion-acoustic envelope solitons in a collisionless unmagnetized EPI plasma (with one-electron temperature distribution), where they found that as the concentration of positrons increases, the instability sets in at lower wave numbers. Later, Mishra *et al.* [23] investigated ion-acoustic double layers in EPI plasma with two-electron temperature distributions. They found that there exist two critical concentrations of positrons, which decide the existence and nature of the ion-acoustic double layers.

The effects of planar and nonplanar (cylindrical and spherical) geometries as well as the positron concentration on the amplitude modulation of ion-acoustic envelope solitary waves in EPI plasma, with two-electron temperature distributions, have not been investigated before. Therefore, we shall investigate the amplitude modulation of the ion-acoustic envelope solitary waves in planar and nonplanar geometries as well as the role of the positron concentration in an EPI plasma with two-electron temperature distributions.

The paper is organized in the following fashion: the basic equations governing the nonlinear dynamics of the ion-acoustic envelope solitary waves are presented and a modified nonlinear Schrödinger equation (NLSE) is derived in Sec. II. In Sec. III, we discuss the modulational instability analysis of the ion-acoustic envelope solitary waves in planar and nonplanar (cylindrical and spherical) geometries under the effects of various physical parameters. Finally, the results are summarized in Sec. IV.

II. DERIVATION OF THE CYLINDRICALLY AND SPHERICALLY MODIFIED NLSE

We consider an unmagnetized plasma with cold ions and two distinct groups of electrons having densities n_{ec} and n_{eh}, and temperatures T_c and T_h, respectively, beside hot positrons. For the ion-acoustic waves, the electrons and positrons inertia are neglected and the electron and positrons fluids can be assumed to be separately in equilibrium with the electrostatic potential φ. Thus the electron number densities are given by

$$n_{ec} = \mu \exp\left(\frac{1}{\mu + \eta\beta}\varphi\right), \quad (1)$$

$$n_{eh} = \eta \exp\left(\frac{\beta}{\mu + \eta\beta}\varphi\right), \quad (2)$$

while for the hot positrons

$$n_p = \exp(-\gamma\varphi), \quad (3)$$

where $\varphi = e\tilde{\varphi}/T_{eff}$, $\mu = n_{ec0}/n_0$, $\eta = n_{eh0}/n_0$, $\beta = T_c/T_h$, $\gamma = T_{eff}/T_p$ with $T_{eff} = T_c/[\mu + \eta\beta]$, and n_{ec0}, n_{eh0}, n_{p0}, and n_0 are the equilibrium densities of two-electron components, positron component, and ion component, respectively. The dynamics of the cold ion fluid is governed by the hydrodynamic equations, namely,

$$\frac{\partial n}{\partial t} + \frac{1}{r^\nu}\frac{\partial}{\partial r}(r^\nu n u) = 0, \quad (4)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} = -\frac{\partial \varphi}{\partial r}. \quad (5)$$

The system of equations is closed by the Poisson equation

$$\frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu \frac{\partial \varphi}{\partial r}\right) = n_h + n_c - \alpha n_p - (1-\alpha)n, \quad (6)$$

where $\nu = 0$, for one-dimensional geometry and $\nu = 1, 2$ for cylindrical and spherical geometries, respectively. Here $\alpha = n_{p0}/n_0$. It may be noticed that in the present model, it is assumed that the positron annihilation time is larger than the inverse of the characteristic frequency of the ion-acoustic wave. Under such conditions, one can assume that annihilation of positrons with electrons is negligible and the effect of annihilation can be neglected. In the above equations, n and u are the density and fluid velocity of the ion species. In Eqs. (1)–(6), the densities have been normalized to the equilibrium value n_0, x to $\lambda_D = (T_{eff}/4\pi n_0 e^2)^{1/2}$, the ion fluid velocity u to the effective ion-acoustic speed $C_s = (T_{eff}/m_i)^{1/2}$, and the time is in the unit of the ion plasma period $\omega_{pi}^{-1} = (4\pi n_0 e^2/m_i)^{-1/2}$. Here, m_i is the ion mass and e is the electronic charge. The charge-neutrality condition is expressed as $\mu + \eta = 1$. Mishra *et al.* [23] studied small amplitude ion-acoustic double layers for the system of Eqs. (1)–(6) when $\nu = 0$.

In order to investigate the amplitude modulation of ion-acoustic envelope solitary waves in our plasma system, we employ the standard reductive perturbation multiple scales technique [25]. The independent variables are stretched as $\xi = \epsilon(r - v_g t)$ and $\tau = \epsilon^2 t$, where ϵ is a small (real) parameter and v_g is the envelope group velocity to be determined later. The dependent variables are expanded as

$$\boldsymbol{\Gamma}(r,t) = \boldsymbol{\Gamma}_0 + \sum_{m=1}^{\infty} \epsilon^m \sum_{L=-\infty}^{\infty} \Gamma_L^{(m)}(\xi,\tau)\exp(iL\theta), \quad (7)$$

where

$$\boldsymbol{\Gamma}_L^{(m)} = [u_L^{(m)} \quad u_L^{(m)} \quad \varphi_L^{(m)}]^T,$$

$$\boldsymbol{\Gamma}_L^{(0)} = [1 \quad 0 \quad 0]^T, \quad \theta = kr - \omega t,$$

k and ω are real variables representing the fundamental (carrier) wave number and frequency, respectively. All elements of $\boldsymbol{\Gamma}_L^{(m)}$ satisfy the reality condition $\boldsymbol{\Gamma}_{-L}^{(m)} = \boldsymbol{\Gamma}_L^{*(m)}$, where the asterisk denotes the complex conjugate. Substituting Eq. (7) into Eqs. (1)–(6) and collecting terms of the same powers of ϵ, the first-order $(m=1)$ equations with $L=1$, give $n_{i_1}^{(1)} = \frac{k^2}{\omega^2}\varphi_1^{(1)}$, $u_{i_1}^{(1)} = \frac{k}{\omega}\varphi_1^{(1)}$, and

$$\omega = \left(\frac{k^2(1-\alpha)}{1+k^2+\alpha\gamma} \right)^{1/2}. \tag{8}$$

The second-order ($m=2$) reduced equations with $L=1$ are given by

$$i\xi(-\omega n_1^{(1)} + k u_1^{(1)}) + i\tau v_g(-\omega n_1^{(2)} + k u_1^{(2)}) = \tau v_g^2 \frac{\partial n_1^{(1)}}{\partial \xi} - \tau v_g \frac{\partial u_1^{(1)}}{\partial \xi},$$

$$-i\omega u_1^{(2)} + ik\varphi_1^{(2)} = v_g \frac{\partial u_1^{(1)}}{\partial \xi} - \frac{\partial \varphi_1^{(1)}}{\partial \xi},$$

and

$$\xi[(1-\alpha)n_1^{(1)} - (1+k^2+\alpha\gamma)\varphi_1^{(1)}] + \tau v_g[(1-\alpha)n_1^{(2)} - (1+k^2+\alpha\gamma)\varphi_1^{(2)}] = -2ik\tau v_g \frac{\partial \varphi_1^{(1)}}{\partial \xi}. \tag{9}$$

Solving the system of equations (9) with the help of the first-order quantities, we can express the second-order quantities with $L=1$ as

$$n_1^{(2)} = \frac{k}{\omega^3}\left[k\omega\varphi_1^{(2)} - 2i(\omega - kv_g)\frac{\partial \varphi_1^{(1)}}{\partial \xi} \right],$$

$$u_1^{(2)} = \frac{1}{\omega^2}\left[k\omega\varphi_1^{(2)} - i(\omega - kv_g)\frac{\partial \varphi_1^{(1)}}{\partial \xi} \right], \tag{10}$$

with the compatibility condition

$$v_g = \frac{\omega(1-\alpha-\omega^2)}{k(1-\alpha)} = \frac{\partial \omega}{\partial k}. \tag{11}$$

Recall that compatibility condition (11) is the group velocity of the envelope soliton.

The second harmonic modes ($m=L=2$) arising from the nonlinear self-interaction of the carrier waves are obtained in terms $[\varphi_1^{(1)}]^2$ as

$$n_2^{(2)} = \Delta_1[\varphi_1^{(1)}]^2,$$

$$u_2^{(2)} = \Delta_2[\varphi_1^{(1)}]^2,$$

and

$$\varphi_2^{(2)} = \Delta_3[\varphi_1^{(1)}]^2, \tag{12}$$

where Δ_1, Δ_2, and Δ_3 are given in the Appendix.

The nonlinear self-interaction of the carrier wave also leads to the creation of a zeroth-order harmonic. Its strength is analytically determined by taking $L=0$ component of the third-order reduced equations which can be expressed as

$$n_0^{(2)} = \Delta_4|\varphi_1^{(1)}|^2,$$

$$u_0^{(2)} = \Delta_5|\varphi_1^{(1)}|^2,$$

and

$$\varphi_0^{(2)} = \Delta_6|\varphi_1^{(1)}|^2, \tag{13}$$

where Δ_4, Δ_5, and Δ_6 are given in the Appendix.

Finally, the third harmonic modes ($m=3$ and $L=1$), with the aid of Eqs. (12) and (13), give a system of equations, which can be reduced to the following modified NLSE:

$$i\frac{\partial \phi}{\partial \tau} + P\frac{\partial^2 \phi}{\partial \xi^2} + Q|\phi|^2\phi + i\frac{\nu}{2\tau}\phi = 0, \tag{14}$$

where $\phi \equiv \varphi_1^{(1)}$ for simplicity. The term $i\frac{\nu}{2\tau}\phi$ in Eq. (14) accounts for the nonplanar (cylindrical and spherical) geometries effects. The dispersion coefficient P reads

$$P = \frac{1}{2}\frac{\partial^2 \omega}{\partial k^2} = \frac{3}{2}\frac{v_g}{\omega}\left(v_g - \frac{\omega}{k} \right), \tag{15}$$

and the nonlinear coefficient Q is

$$Q = -\frac{1}{2}(\omega\Delta_1 + 2k\Delta_2) + \frac{1}{4}(\omega\Delta_4 + 2k\Delta_5) - \frac{(2\Delta_3 - \Delta_6)\omega^3[2\alpha\gamma^2\mu\eta\beta + \mu(\alpha\gamma^2\mu - 1) + \eta\beta^2(\alpha\gamma^2\eta - 1)]}{4k^2(1-\alpha)(\beta\eta + \mu)^3} + \frac{\omega^3\{\mu + \eta\beta^3(1 + \alpha\eta^2\gamma^3) + \alpha\mu\gamma^3[\mu^2 + 3\beta\eta(\beta\eta + \mu)]\}}{4k^2(1-\alpha)(\beta\eta + \mu)^3}. \tag{16}$$

III. STABILITY ANALYSIS AND DISCUSSION

A. Derivation of the nonlinear dispersion relation

To investigate the stability/instability of the planar and nonplanar envelope IA waves, we consider the development of the small modulation $\delta\phi$ according to

$$\phi = [\bar{\phi}_0 + \delta\phi(\xi,\tau)]\exp\left[-i\int_{\tau_0}^{\tau} \Delta(\tau')d\tau' - \frac{\nu}{2}\ln \tau \right], \tag{17}$$

where $\bar{\phi}_0$ is the constant (real) amplitude of the pump carrier wave and Δ is a nonlinear frequency shift, and taking the perturbation $\delta\phi$ as

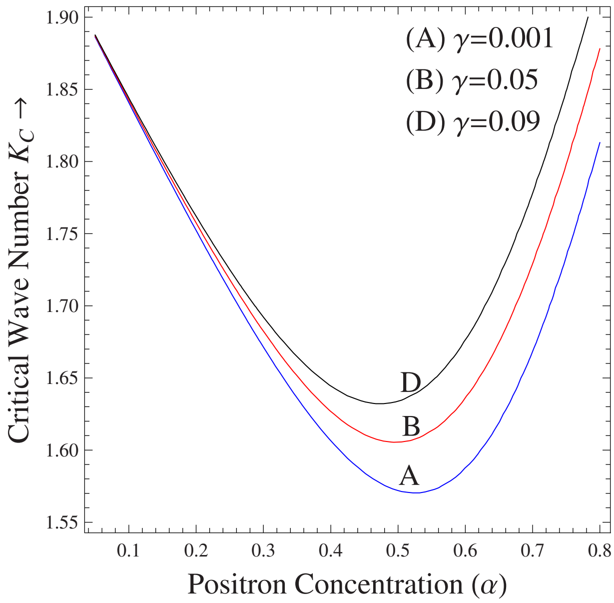

FIG. 1. (Color online) Variation in the critical wave number (K_c) against the positron concentration (α) with μ=0.01 and β=0.05 for different values of γ.

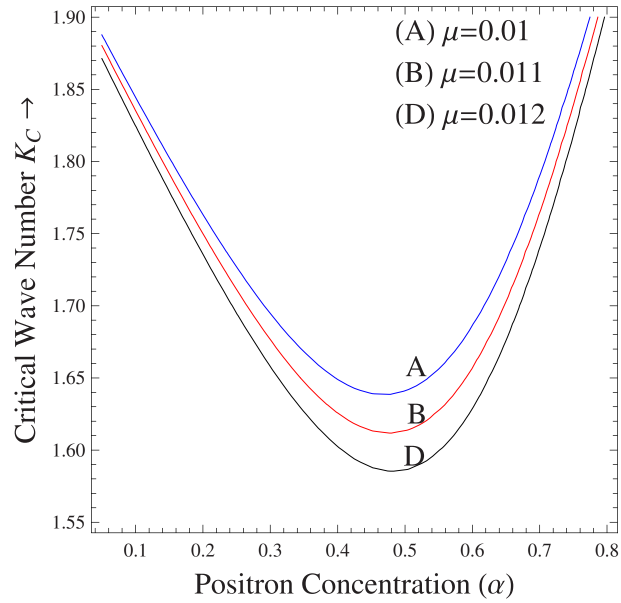

FIG. 2. (Color online) Variation in the critical wave number (K_c) against the positron concentration (α) with γ=0.1 and β=0.05 for different values of μ.

$$\delta\phi = \delta\bar{\phi}_0 \exp\left[i\left(K\xi - \int_{\tau_0}^{\tau} \Omega(\tau')d\tau' \right)\right] + \text{c.c.}, \qquad (18)$$

where $K\xi - \int_{\tau_0}^{\tau}\Omega d\tau'$ is the modulation phase with K and Ω as the perturbation wave number and frequency of the modulation, respectively. Using Eqs. (17) and (18) into Eq. (14), one obtains the nonlinear dispersion relation [26]

$$\Omega^2 = (PK^2)^2\left(1 - \frac{Q}{P}\frac{2|\bar{\phi}_0|^2}{\tau^{\nu}}\frac{1}{K^2}\right), \qquad (19)$$

which exactly reduces to the dispersion relation for the planar geometry when ν=0. We immediately see that the modulation instability condition will be satisfied if $PQ>0$ and $K^2 \le K_c^2(\tau) = 2Q|\bar{\phi}_0|^2/(P\tau^{\nu})$.

B. Stability/instability of planar envelope excitations

To investigate the modulational stability/instability of the planar envelope pulses, one sets ν=0 into the nonlinear dispersion relation (19). It is straightforward to see that a negative sign for PQ is required for wave amplitude (modulational) stability. On the other hand, a positive sign of PQ allows for a random perturbation of the amplitude to grow and may thus lead to wave collapse or blowup.

To investigate the stability profile, we have determined in various regimes the critical wave number threshold K_c (at which PQ=0), which indicates where the instability sets in. The variation in the critical wave number K_c with respect to the positron concentration α is shown in Fig. 1 for given values of μ and β, and three different values of γ. It is clear that increasing the positron concentration (α) decreases the critical wave number (K_c) until α approaches certain value α_c, then further increase in α beyond α_c increases the value of K_c. Such behavior reflects the existence of double layers. Also, an increase in γ increases the critical wave number K_c and decreases the value of α_c.

The effect of μ and α on the variation of the critical wave number K_c is depicted in Fig. 2. It is found that the critical wave number (K_c) decreases by increasing α until α approaches α_c, then further increase in α increases the value of K_c, which is the same behavior for Fig. 1. However, the effect of μ is opposite to that of γ, where an increase in μ decreases the critical wave number K_C but increases the value of α_C.

Finally, an estimation of the effect of β (i.e., T_C/T_h) and α on the variation of the critical wave number K_c is shown in Fig. 3. For $\alpha<\alpha_c$ the critical wave number K_c is a decreasing function in α. While K_c is an increasing function in α, for $\alpha>\alpha_c$. An increase in the β value decreases the critical wave number K_c, for $\alpha<\alpha_c$. While, increasing β increases K_c, for $\alpha>\alpha_c$.

C. Stability/instability of spherical and cylindrical excitations

The local instability growth rate, for $\nu\neq 0$, of the nonlinear dispersion relation (19) is given by [26]

$$\text{Im}\,\Omega = PK^2\left(\frac{2Q}{PK^2}\frac{|\bar{\phi}_0|^2}{\tau^{\nu}} - 1\right)^{1/2}. \qquad (20)$$

The instability growth will cease for cylindrical geometry (ν=1) when

$$\tau \ge \tau_{\max} = \frac{2|\bar{\phi}_0|^2}{K^2}\frac{Q}{P}, \qquad (21)$$

and for spherical geometry (ν=2) when

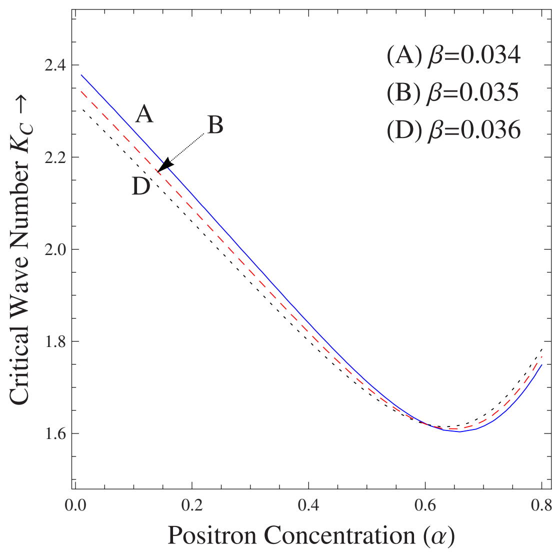

FIG. 3. (Color online) Variation in the critical wave number (K_c) against the positron concentration (α) with γ=0.1 and μ=0.01 for different values of β.

$$\tau \geq \tau_{\max} = \frac{|\bar{\phi}_0|}{K}\sqrt{\frac{2Q}{P}}. \qquad (22)$$

It is clear that there is a modulation instability period (τ) for the cylindrical and spherical wave modulation, which does not exist in the one-dimensional case. The growth (Γ) of the modulation during the unstable period [26] is

$$\Gamma = \exp\left(\int_{\tau_0}^{\tau_{\max}} \text{Im }\Omega d\tau'\right) = \exp\left(\frac{Q|\bar{\phi}_0|^2}{\tau^{\nu-1}} f(R)\right), \qquad (23)$$

where $R=[2Q|\bar{\phi}_0|^2/(PK^2\tau_0^{\nu})]\geq 1$. For the cylindrical geometry, we have

$$f(R) \equiv f_{\text{cyl}} = \arctan\sqrt{R-1} - \frac{\sqrt{R-1}}{R}, \qquad (24)$$

while for the spherical geometry

$$f(R) \equiv f_{\text{sph}} = \frac{1}{R}\left[\sqrt{R}\ln\left(\frac{\sqrt{R}+\sqrt{R-1}}{\sqrt{R}-\sqrt{R-1}}\right) - 2\sqrt{R-1}\right]. \qquad (25)$$

We note that f_{cyl} is an increasing function in R, and $f_{\text{cyl}} \to \pi/2$ as $R\to\infty$. This means that during the modulation instability period, the total growth increase as R does for the cylindrical case. But, for f_{sph} there is a maximum value

$$\max f_{\text{sph}} = \frac{2\sqrt{R_c-1}}{R_c}, \qquad (26)$$

where R_c is determined by

$$4\sqrt{R_c-1} = \sqrt{R_c}\ln\left(\frac{\sqrt{R_c}+\sqrt{R_c-1}}{\sqrt{R_c}-\sqrt{R_c-1}}\right). \qquad (27)$$

For spherical geometry, the modulation instability growth rate will achieve its maximum at $R=R_c$ and then decreases as R increases further. It should be noted that the modulation instability period given by Eq. (21) for cylindrical geometry is longer than that determined by Eq. (22) for spherical geometry; meanwhile, during the unstable period, the modulation instability growth rate is always an increasing function of R in the cylindrical geometry, but not in the spherical geometry, as depicted in Fig. 4. This suggests that the spherical waves are more structurally stable to perturbations than the cylindrical waves.

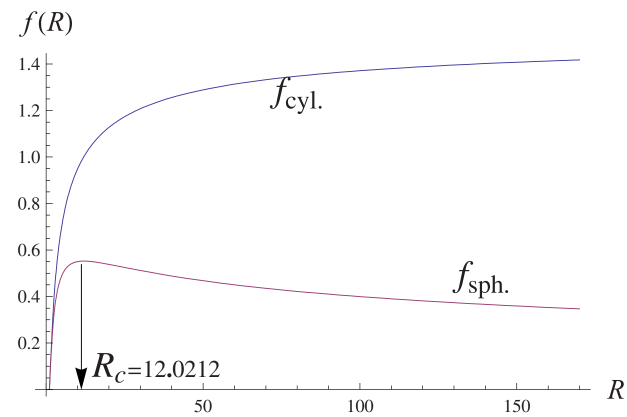

FIG. 4. (Color online) Variation in $f(R)$ against R for cylindrical and spherical geometries given by Eqs. (24) and (25), respectively.

IV. CONCLUSIONS

To summarize, we have investigated the modulational instability of the envelope ion-acoustic solitary waves in an unmagnetized electro-positron-ion plasma for planar as well as for cylindrical and spherical geometries. The critical wave number threshold K_c, which indicates where the instability sets in, has been determined for various regimes. The present study shows that the existence of two-electron temperature distributions in electron-positron-ion plasmas introduces unique features for the nonlinear wave modulation which do not exist in ordinary electron-positron-ion plasmas. It is found that increasing the positron concentration α leads to a decrease in the critical wave number (K_c) until α approaches certain value α_c (critical positron concentration), then further increase in α beyond α_c increases the value of K_c. Such behavior reflects the existence of double layers.

Also, it is found that there is a modulation instability period for the cylindrical and spherical wave modulation, which does not exist in the one-dimensional case. During the unstable period, the modulation instability growth rate is always an increasing function of R in the cylindrical geometry, but not in the spherical geometry. This suggests that the spherical waves are more structurally stable to perturbations than the cylindrical waves.

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APPENDIX: COEFFICIENTS OF THE HARMONIC MODES

The coefficients Δ_j (i.e., $j=0,1,\ldots,6$) are

$$\Delta_0 = 2\alpha\gamma^2\mu\eta\beta + \mu(\alpha\gamma^2\mu - 1) + \eta\beta^2(\alpha\gamma^2\eta - 1), \tag{A1}$$

$$\Delta_1 = \frac{k^2[3k^2(4k^2+\alpha\gamma+1)(\beta\eta+\mu)^2 + \omega^2\Delta_0]}{2(\beta\eta+\mu)^2\omega^2[(4\omega^2+\alpha-1)k^2+(\alpha\gamma+1)\omega^2]}, \tag{A2}$$

$$\Delta_2 = \frac{k}{2\omega^3}\left\{k^2 + \frac{\Delta_0\omega^4 - 3k^4(\alpha-1)(\beta\eta+\mu)^2}{(\beta\eta+\mu)^2[(4\omega^2+\alpha-1)k^2+(\alpha\gamma+1)\omega^2]}\right\}, \tag{A3}$$

$$\Delta_3 = \frac{\Delta_0\omega^4 - 3k^4(\alpha-1)(\beta\eta+\mu)^2}{2(\beta\eta+\mu)^2\omega^2[(4\omega^2+\alpha-1)k^2+(\alpha\gamma+1)\omega^2]}, \tag{A4}$$

$$\Delta_4 = \frac{2(\alpha\gamma+1)(\beta\eta+\mu)^2 v_g k^3 + \omega[k^2(\alpha\gamma+1)(\beta\eta+\mu)^2 + \Delta_0\omega^2]}{(\beta\eta+\mu)^2\omega^3[(\alpha\gamma+1)v_g^2+\alpha-1]}, \tag{A5}$$

$$\Delta_5 = \frac{\omega[k^2(\alpha\gamma+1)(\beta\eta+\mu)^2 + \Delta_0\omega^2]v_g - 2k^3(\alpha-1)(\beta\eta+\mu)^2}{(\beta\eta+\mu)^2\omega^3[(\alpha\gamma+1)v_g^2+\alpha-1]}, \tag{A6}$$

and

$$\Delta_6 = \frac{-2(\alpha-1)(\beta\eta+\mu)^2 v_g k^3 - (\alpha-1)(\beta\eta+\mu)^2\omega k^2 + \Delta_0\omega^3 v_g^2}{(\beta\eta+\mu)^2\omega^3[(\alpha\gamma+1)v_g^2+\alpha-1]}. \tag{A7}$$

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